This paper has been published in Proceedings of Radiovetenskap och kommunikation 99 (RVK 99), *pages 303–307, Karlskrona, Sweden, June 1999.*

DMT PAR-reduction by weighted cancellation waveforms

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Abstract

DMT (discrete multitone) signals have a high peak-to-average power ratio (PAR). In the transmitters, the PAR governs the necessary resolution of the digital-to-analog converter (DAC) and is an important factor for the power consumption of the line-driver.

In a recent paper, we presented a low complexity PAR-reduction method for DMT-transmitters. Here we improve a central part of this method. Computer simulations show a small but clear clip rate and clip power performance improvement with almost no increase in complexity.

1 Introduction

The samples of a discrete multitone (DMT) or orthogonal frequency division multiplexing (OFDM) signal have a nearly Gaussian amplitude distribution. Thus, it has a high *peak-to-average power ratio*[1] (PAR).

Figure 1 shows the parts of the transmitter that are of importance for PAR-reduction. The high PAR is mainly a problem in the digital-to-analog converter (DAC) and in the line-driver.

The DAC clips all samples that exceed a certain maximum amplitude, the *clip level*. If the PAR is reduced, a lower clip level is needed to reach a desired clip rate. Consequently, the prefered quantization of the IFFT output can be obtained with a lower precision, and cheaper, DAC.

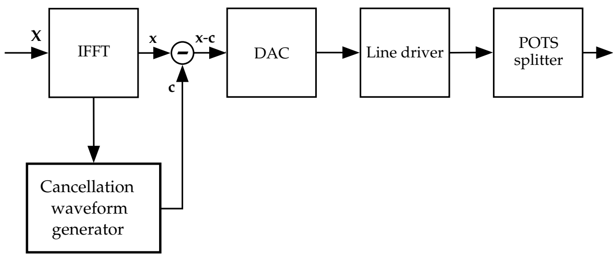


Figure 1: A DMT transmitter.

The *line-driver* amplifies the DAC output and applies it to the line. In our target system, it must be a class A amplifier. Therefore, the power consumption is determined by the range over which the line driver must provide linear amplification, that is, by the amplitude of the highest peaks. Reducing the PAR reduces this range, and thus also the power consumption. This further reduces the cost of the modem.

The results reported in this paper are off-spring of the Telia Research and ST Microelectronics (former SGS-Thomson) joint development of a DMT-VDSL (very high bit-rate digital subscriber loop) system using the Zipper [1, 2, 3] duplex method. In [4] we introduced a low-compexity PAR-reduction method, which is developed for use in this system. The PAR is reduced by adding a waveform that is bandlimited to certain *peak reduction tones*, which consequently cannot be used for data transmission. A special circuit, the *cancellation waveform generator*, constructs this waveform so that it has sharp and unique peak(s) in counterphase to the largest peak(s) of the DMT frame. Our method is based on an iterative method [5, 6, 7, 8, 9], but with additional restrictions and refinements in order to adapt it to our target system.

The main characteristics of the approach in [4] are

*Partially supported by TFR (Teknikvetenskapliga Forskningsrådet, Sweden).

**Partially supported by the Austrian Science Foundation FWF under grant S 7001-MAT.

[1]The peak-to-average power ratio is defined as the ratio $\|\mathbf{x}\|_\infty^2/\sigma^2$ of the maximum absolute value squared of a DMT frame $\mathbf{x} = (x_1, x_2, \ldots, x_{2N})$ to its expected mean power σ^2.

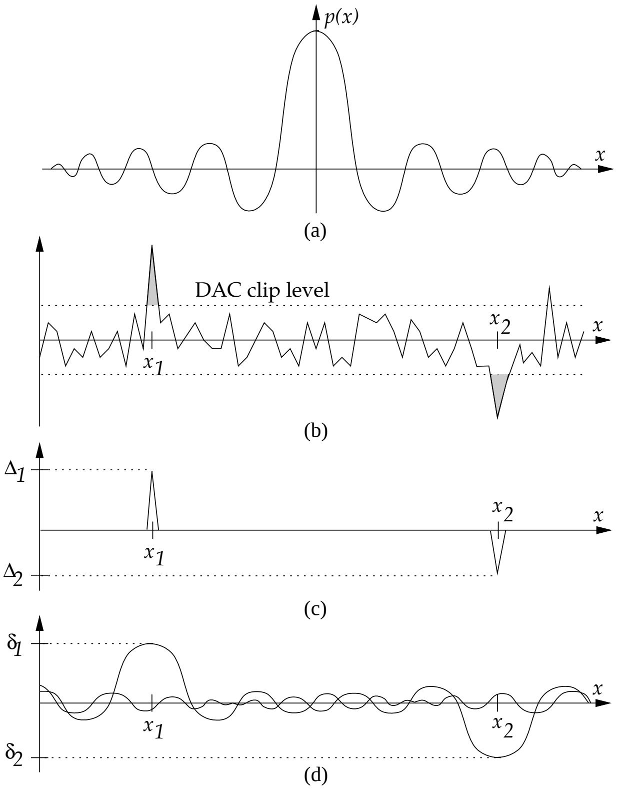


Figure 2: Weighted peak reduction: (a) The precalculated and stored peak-cancellation kernel; (b) the DMT signal; (c) identified potential clips; (d) the translated and rescaled peak-cancellation kernels.

that we *avoid iterations,* due to the low latency budget in the VDSL requirements. We also *replace multiplications with bit-shifts* in the necessary computations. This reduces the complexity considerably, with no significant change in performance. We use *only the first 60 tones* for peak reduction, thus maintaining the data rate. This frequency band cannot be used for data transmission, since it is occupied by POTS (plain old telephone service) and therefore filtered out in the POTS splitter. It can, however, be used to reduce the PAR in the DAC and the line driver. In [4], we also give special attention to when several peaks are reduced simultaneously. This was done by reducing the peaks down to a certain peak reduction level, slightly below the clip level. In the present paper, we introduce a more sophisticated solution, *weighted peak reduction.*

In Section 2 we present the new algorithm, the simulation results are given in Section 3. An interesting result is that weighting almost completely removes the (clip rate) performance loss that comes from replacing multiplications with bit-shifts.

For further references to PAR-related papers the reader may consult [4], which also contains a more detailed explanation of the target system and the implementation.

2 Weighted peak reduction

Our method tries to reduce each peak to a fixed *peak reduction level*, slightly below or equal to the clip level. Figure 2 shows the steps needed to reduce two clips per DMT frame. A precalculated waveform, the *peak-cancellation kernel*, is stored in a memory. It is bandlimited to the first 60 subcarriers. A simple programmable processor, or a dedicated circuit, identifies potential clips in the IFFT output, as illustrated in Figure 2(b) and (c). Note that Δ_i is the distance down to the peak reduction level. For each peak, the cancellation waveform generator (parallelly) scales and subtracts one copy of the stored waveform from the DMT-signal. With notation as in the figure, the amplitudes δ_i are computed such that the constructed waveform has amplitude Δ_i at x_i. This is done in equation (3) below.

Our new algorithm for reducing up to M peaks per frame is:

1. Locate the (at most) M highest amplitude samples that exceed the clip level. (This could be replaced by finding the at most M *first* such peaks.)
2. Find the position x_i of each such peak and the amplitude Δ_i by which it exceeds the peak reduction level.
3. Compute the reduction amplitudes δ_i. In the case of bit-shift scaling, each δ_i is then rounded upwards to an even power of two.
4. Compute the sum of scaled copies of the peak-cancellation kernel p.
5. Subtract the generated waveform from the IFFT output.

In this algorithm, we want to choose δ_j such that

$$\Delta_i = \sum_{j=1}^{M} \delta_j \cdot p[x_i - x_j], \tag{1}$$

where p is read with circular adressing from the memory. With the notation $\boldsymbol{\Delta} \stackrel{\text{def}}{=} (\Delta_1, \ldots, \Delta_M)^T$ and $\boldsymbol{\delta} \stackrel{\text{def}}{=} (\delta_1, \ldots, \delta_M)^T$ we write the linear system (1) in matrix form

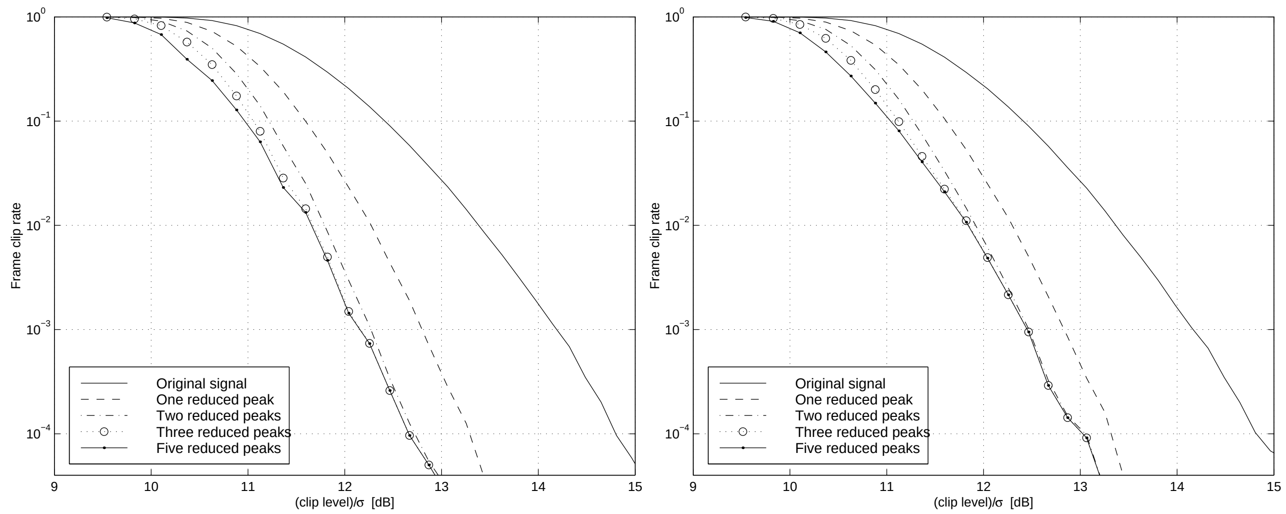
$$\boldsymbol{\Delta} = W\boldsymbol{\delta},$$

where

$$(W)_{i,j} = p[x_i - x_j]. \tag{2}$$

Thus a solution is given by

$$\boldsymbol{\delta} = W^{+}\boldsymbol{\Delta}, \tag{3}$$



(a) (b)

Figure 3: Frame clip rate plotted against the clip level (divided by the signal's standard deviation σ): (a) Weighting; (b) Weighting and bit-shift scaling.

where W^+ denotes the pseudoinverse [10] of W. If, for example, $M = 2$ and p is symmetric with maximum amplitude $a = |p[0]|$, then $W = \left(\begin{smallmatrix} a & b \\ b & a \end{smallmatrix}\right)$ with $|b| < |a|$. Thus, W is invertible and $W^+ = W^{-1}$. This is the case (when $M = 2$) for the sinc-function used in our simulations.

The peak reduction level was introduced in [4] as a safety margin against the interaction between translated waveforms. There, we concluded that a good choice of this level is 98% of the clip level, and this gave a clear performance improvement.

However, this is not the best safety margin for all frames. On the one hand, when a frame has only one potential clip, then no safety margin is needed. Instead, the higher waveform amplitude increases the probability of creating new clips. On the other hand, sometimes this safety margin can be too small to get all clips reduced. The weighting is a way around this problem of a fixed safety margin. The reduction amplitudes are always chosen so that the peaks are reduced exactly to the peak reduction level, which then can be set equal to the DAC clip level. Moreover, when there are many ways to do this, then the pseudoinverse always gives the minimum 2-norm solution [10], that is, the vector $\boldsymbol{\delta}$ that has minimum length.

	Multiplicative	Bit-shift
No weighting	2.0 dB	1.7 dB
Weighting	2.2 dB	1.9 dB

Table 1: Clip level reduction when reducing (at most) two peaks with a fixed frame clip rate of 10^{-4}.

3 Computer simulation results

We have examined the performance of our method with MATLAB simulations. The first 60 DMT subcarriers (of 2048) are used for peak reduction. All the remaining subcarriers are bit-loaded [11] for an 800 m long TP2 cable [12]. The signal constellations used are QAM for an even number of bits, and n-cross, 2-PAM or 8-square for an odd number of bits[2]. Round-off errors in the DAC are not simulated.

The used peak approximation waveform is a sinc-function. Other choices of this waveform, for example as proposed in [6], might give some further performance improvement.

The simulated peak cancellation generator tries to reduce the largest peak(s) that exceed the clip level.

The number of simulated frames (ordered as in Figure 4) is (a) 175 400, (b) 197 400, (c) 238 800 and (d) 175 200.

3.1 Clip rate

Given a fixed frame clip rate of 10^{-4}, Table 1 summarizes the clip rate performance shown in Figure 3 and the corresponding figures in [4]. It shows the clip level improvement when reducing up to two peaks per frame. Reducing (at most) one peak gives 1.5 dB reduction in all cases.

The peak reduction level is set to 98% of the clip level, except when weighting is combined with multiplicative scaling. Then setting the levels equal gave

[2]Thus the simulations take into consideration any effects that might arise because the tail of the distribution is not exactly Gaussian.

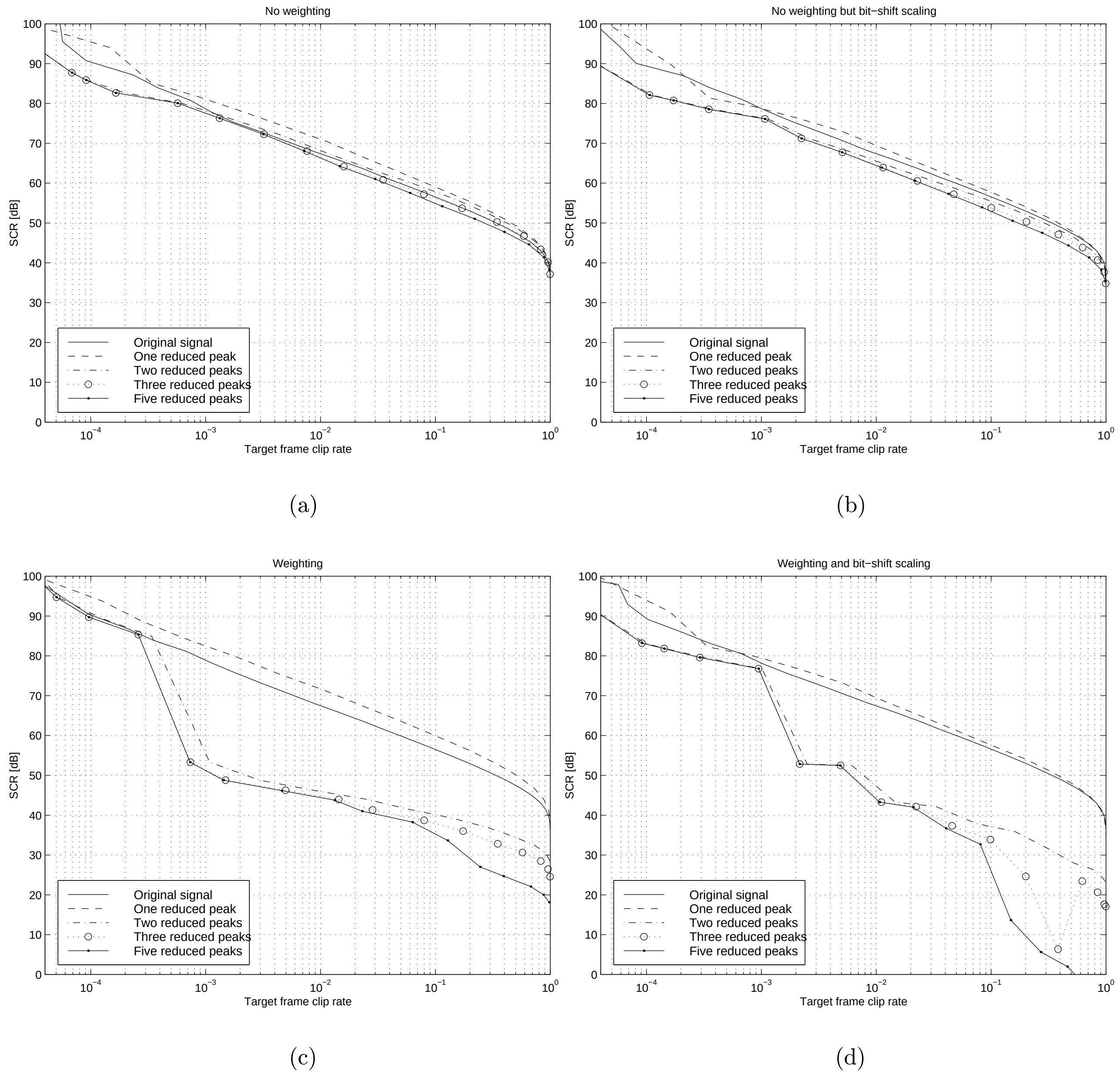


Figure 4: SCR in a system with a fixed target clip rate. Weighting gives a clear but small performance improvement at clip rates interesting in our system. (Comparisons between different number of reduced peaks require, for example, a fixed clip level.) The plots are ordered as in Table 1: (a) multiplicative scaling; (b) bit-shift scaling; (c) weighting and multiplicative scaling; (d) weighting and bit-shift scaling.

better results. This was expected, since weighting (without bit-shift scaling) removes the need for an extra safety margin.

Table 1 shows a small, but clear performance improvement. Note that the performance loss that follows from applying bit-shift scaling is almost completely removed by the weighting. Thus the combination of weighting and bit-shift scaling gives a large reduction of the complexity with almost no change of the clip rate performance.

3.2 Clip power

The signal to clip noise power ratio (SCR) depends on the clip level, which is chosen so that the frame clip rate after peak reduction meets the system requirements. Figure 4 shows the SCR plotted against the required frame clip rate.

At the frame clip rates interesting for our target system (about 10^{-4} and below), PAR-reduction with weighting performs slightly better than the same method without weighting. This is a fair comparison, since the weighted method achieves this clip rate at a slightly *lower* clip level (see Table 1), with the additional benefits described in Section 1. Comparisons at a fixed clip level show that reducing several peaks reduces the clip power up to 10 dB extra, compared to reducing (at most) one peak.

At higher clip rates (not interesting for our target system), weighting increases clip power with no large change of the clip level. This suggests that in this case[3] weighting sometimes produces higher amplitude waveforms when the peak reduction fails. Therefore, a possible subject for further investigation is to introduce a threshold to determine when to use the peak reduced signal. For example, if peak reduction fails with a high amplitude waveform, then it could be better to use the original IFFT output. This could improve the clip power performance with no change in clip rate performance, but such an investigation is out of the scope of this paper.

3.3 Complexity

The presented results suggest choosing $M = 2$, since larger M only gives a very small further improvement. For such small M, the complexity of the weighting-computations (2) and (3) is negligible, and the total complexity is roughly $2N$ comparisons, $2MN$ additions, and $2MN$ bit-shifts, as computed in [4].

4 Summary and conclusions

We have showed that our weighted PAR-reduction method gives a small but clear performance improvement compared to the method presented in [4], and with almost no increase in complexity.

One interesting result is that the combination of weighting and bit-shift scaling gives almost the same clip rate performance as the much more complex case of no weighting and multiplicative scaling.

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[3] The plots show this behaviour when the frame clip rate without peak reduction is of the order 10^{-1} or higher.