 This paper has been published in Proceedings of the 5th International Symposium on Digital Signal Processing for Communication Systems (DSPCS'99), *pages 164–169, Perth, Australia, Feb. 1999*

A low-complexity PAR-reduction method for DMT-VDSL

Per Ola Börjesson[a], Hans G. Feichtinger[b,*], Niklas Grip[c], Mikael Isaksson[d],
Norbert Kaiblinger[b,*], Per Ödling[a], Lars-Erik Persson[c]

[a] Department of Applied Electronics, Lund Institute of Technology, Box 118, SE-221 00 Lund, Sweden.

[b] NUHAG (Numerical Harmonic Analysis Group), Dept. of Mathematics, University of Vienna, Strudlhofgasse 4, A-1090 Vienna, Austria.

[c] Department of Mathematics, Luleå University of Technology, SE-971 87 Luleå, Sweden.

[d] Telia Research AB, Aurorum 6, SE-977 75 Luleå, Sweden.

Abstract

DMT-VDSL signals have a high peak-to-average power ratio (PAR). In the transmitters, the PAR governs the necessary resolution of the digital-to-analog converter (DAC) and is an important factor for the power consumption of the line-driver. Aiming at implementation in a specific system, we propose a low complexity PAR-reduction method based on the iterative algorithm derived in [5, 20, 24, 25]. We maintain good performance while stressing a straigthforward and low-complex implementation. Key elements of the method are: low latency; no loss in data rate; precalculated and stored peak-cancellation waveform; and bit-shifts (multiplication with powers of two) replacing the scaling of the waveform.

Computer simulations show that, for a DMT frame length of 4096 samples and a frame clip rate of 10^{-4}, the PAR can be reduced about 1.5 to 2.0 dB, depending on the number of peaks cancelled. When multiplication is replaced by bit-shifts, the reduction is still 1.5–1.7 dB.

1 Introduction

The samples of a discrete multitone (DMT) or orthogonal frequency division multiplexing (OFDM) signal have a nearly Gaussian amplitude distribution. Thus it has a high *peak-to-average power ratio*[1] (PAR). We will propose and discuss a PAR reduction method in the context of a VDSL (very high bit-rate digital subscriber loop) system that is currently being designed by Telia Research AB and ST Microelectronics (former SGS-Thomson).

The high PAR of the DMT signal has to be considered when setting the range of the digital-to-analog converter (DAC) and dimensioning the line-driver. We will focus on reducing the problem that the high PAR causes for the DAC. Our proposed PAR-reduction device is placed between the IFFT and the DAC, as shown in Figure 1 (a).

The DAC clips all samples that exceed a certain maximum amplitude, the *clip level*. Setting this level is a trade-off between clipping probability and quantization noise level: decreasing the clip level will increase the average clip noise but decrease the quantization noise. It is usually set so that the total SNR is minimized [10]. A lower PAR will increase the SNR or allow for a DAC with lower resolution to be used.

The line-driver is amplifying the DAC output to the average power required by the application in question, here VDSL. It should provide linear amplification up to the amplitude of the highest peaks of the signal. If this amplitude is brought closer to the standard deviation of the signal, a line-driver with lower power consumption can be used.

All this motivated the recent development of an efficient iterative PAR-reduction method for DMT, independently derived by Tellado and Cioffi [20, 24, 25] and Gatherer and Polley [5]. The basic idea is to add a waveform to the DMT-signal. The iterative algorithm constructs this waveform so that it has sharp and unique peaks in counterphase to the largest peak(s) of each DMT frame. This waveform is bandlimited to a certain set of DMT tones, the *peak-reduction tones*, which carry no data. Since the use of inband tones would reduce the data rate, Gatherer and Polley [5] suggested to only use tones which carry no data because of their low SNR. Tellado and Cioffi [20, 24, 25] investigated how the PAR-reduction performance improves with a good choice of peak reduction tones and with an increasing number of iterations. Moreover, they constructed sharper peaks and improved the performance considerably by also using some inband tones for peak reduction (at the cost of a reduced data rate). Furthermore, they introduced an algorithm which finds a good choice of peak reduction tones. In [22],

*The second and fifth named authors have been partially supported by the Austrian Science Foundation FWF under grant S 7001-MAT.

[1]The peak-to-average power ratio is defined as the ratio $\|\mathbf{x}\|_\infty^2/\sigma^2$ of the maximum absolute value squared of a DMT frame $\mathbf{x} = (x_1, x_2, \ldots, x_{2N})$ to its expected mean power σ^2.

Tellado and Cioffi developed a related method (without data rate loss), based on constellation expansion. Both methods are described in [23], and covered in more detail in [19]. Other PAR-related results can be found in [2, 3, 4, 6, 8, 9, 11, 12, 13, 14, 15, 16, 21, 27, 28, 29].

The results reported in this paper are off-spring of Telia Research's and ST Microelectronic's joint development of a VDSL-system. The PAR-investigation is the first result of a cooperation between researchers from Universities in Sweden and Austria, and based on ideas of implementation from Telia Research AB.

2 The intended target system

This investigation was done in conjunction with the development of a VDSL-system. Our PAR-reduction has been developed for use in this system, but many of the results are general or are to be adapted according to the circumstances. Our intended target system is a DMT-VDSL system using the Zipper [7, 17, 18] duplex method. The most important characteristic affecting the PAR-reduction is maybe the high number of subcarriers, 2048. This gives very long DMT-symbols. For latency reasons (the latency budget for the transmission layer in the emerging VDSL standard is small) iterative PAR-reduction is possible only for a method with low complexity. Thus, a natural first step is to try to reduce the complexity of a one shot, on-the-fly PAR-reducer.

Figure 1 (a) shows where in the system the PAR-reducing circuit is located. (The figure only shows details that are of importance for PAR-reduction.) The IFFT-output in Figure 1 (a) is the DMT-signal with the high peaks. The DMT-signal is illustrated in Figure 1 (b). The first problems arise in the digital-to-analog converter, where clipping of the signal can occur. The POTS (plain old telephone service) splitter separates the VDSL signal in frequency from standard telephone signals and ISDN signals. As it is the lowermost part of the spectrum that is used for these services, the POTS-splitter is a high-pass filter as seen from the VDSL-system. The exact cut-off frequency of the filter varies depending on *e.g.* if ISDN is present. Here we have assumed that the first 60 subcarriers will be filtered out and cannot be used for data transmission. However, they can be used for PAR-reduction.

Figure 2 describes more in detail the structure of the PAR-reducer. A precalculated waveform (which we will refer to as the peak-cancellation kernel) is stored in a memory. It has a reasonably sharp peak and (here) occupies the first 60 subcarriers. A simple programmable processor, or a dedicated circuit, identifies peaks in the DMT-signal that would be clipped. For each peak it (parallelly) scales and subtracts one copy of the stored waveform from the DMT-signal. The peak-cancellation is described in greater detail in Section 3.

3 A non-iterative PAR-reduction method

The objectives for the PAR reducing device is to add a waveform with a sharp and unique peak in counterphase to the largest peak of the DMT frame. Similarly, more than one copy of the waveform can be added to cancel more than one peak. However, care should be taken not to create new peaks in the process.

The iterative peak-reduction algorithm uses a *peak-cancellation kernel* $\mathbf{p}$, which is a waveform with an approximate impulse. This kernel is bandlimited to certain *peak reduction tones*, which are not used for data transmission. In an effort to implement PAR-reduction in an actual VDSL system, we add some restrictions and refinements to the iterative method suggested in [5, 20, 24, 25]:

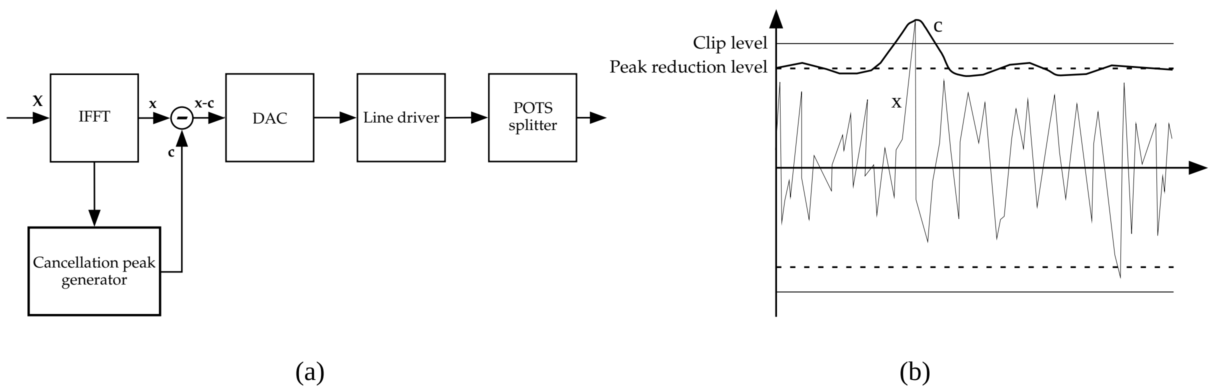


Figure 1: Peak reduction in a DMT transmitter: (a) The proposed peak reduction method reduces the highest peak(s) of each DMT frame by subtracting bandlimited and scaled approximations of the potential clips; (b) the signal in the center is the original DMT-signal containing a peak that would be clipped. This clip is eliminated by subtracting the shifted and scaled peak cancellation kernel indicated in the figure.

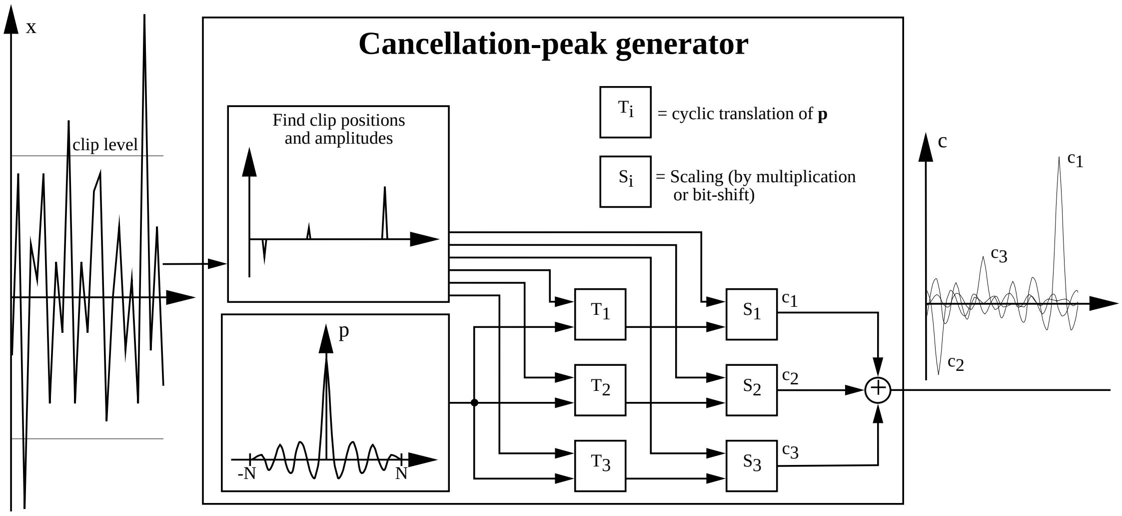


Figure 2: The cancellation-peak generator first finds the M largest peaks of the current DMT frame. (In this figure, $M = 3$.) The peak-cancellation kernel $\mathbf{p}$ is stored in a memory. M copies are cyclically read from the memory and scaled to produce approximations of the M largest clips. These are added to produce the output $\mathbf{c}$, which is the waveform to be subtracted from the DMT-signal. If the peak-reduction level is set lower than the clip-level, the amplitudes of c_1, c_2 and c_3, are larger than the actual clips.

We avoid iterations. The latency allowed by the VDSL requirements for the physical layer does not allow for the PAR-reduction to introduce much latency. For instance the buffering of data and the processing time affect the latency.

By replacing multiplications with bit-shifts (multiplication/division with powers of two), we reduce the complexity considerably. Surprisingly, there is no significant change in performance. (See Section 4.2.)

We use only the first 60 tones for peak reduction, thus maintaining the data rate. This frequency band is used by POTS and therefore filtered out in the POTS splitter. It can, however, be used to reduce the PAR in the DAC and the line driver, which are placed between the IFFT and the POTS splitter, see Figure 1 (a).

The highest peaks are reduced down to a fixed peak reduction level. We find good choices of this level, in the sense that we aim to get the highest peak below the DAC clip level in as many DMT frames as possible, and in only one iteration. (Thus we do *not* attempt to come as close as possible to minimal PAR in a few iterations.)

We give special attention to when several peaks are reduced simultaneously in Section 4.1.

These requirements are met by the cancellation-peak generator depicted in Figure 2. It reduces up to M peaks in each frame. The algorithm is:

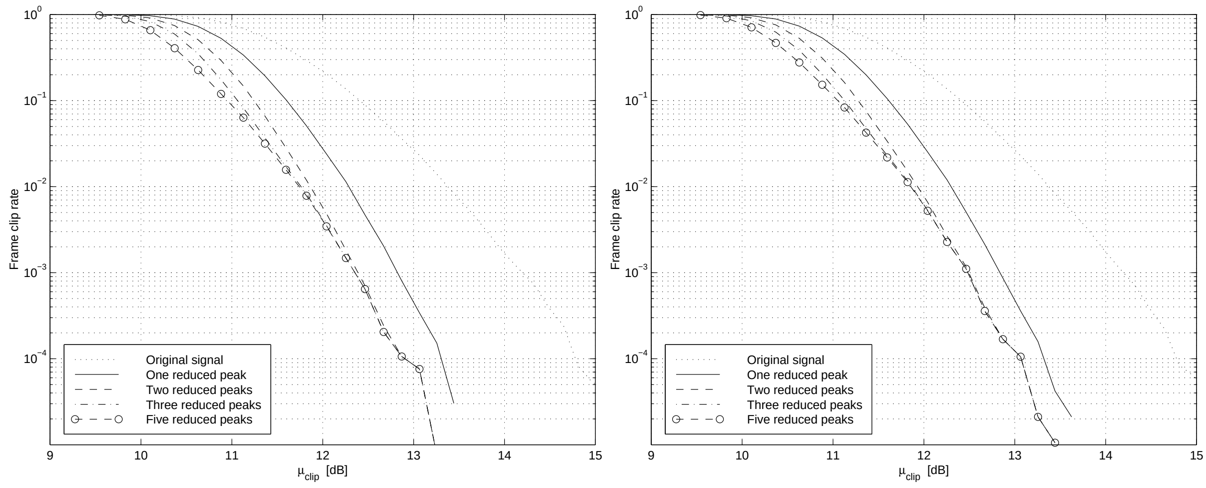
1. Locate the $\leq M$ highest amplitude samples that exceed the clip level. (This could be replaced by finding the $\leq M$ first such peaks.)

2. Compute the amplitude Δ_i by which each such peak exceeds the predefined *peak reduction level*, see Figure 1 (b).

3. Subtract scaled copies of $\mathbf{p}$ from $\mathbf{x}$. In the low-complexity implementation of Section 4.3, the multiplication by Δ_i is replaced by by bit-shifting.

4 Computer simulation results

We have used computer simulations to verify the performance of our method. In our simulations, the first 60 DMT subcarriers (of 2048) are used for peak reduction. All the remaining subcarriers are bit-loaded [26] for an 800 m long TP2 cable [1]. The signal constellations used are QAM for an even number of bits, and n-cross, 2-PAM or 8-square for an odd number of bits. We have used a sinc-function as the peak approximation waveform. Shaping of



(a) (b)

Figure 3: Frame clip rates before and after peak reduction: (a) multiplicative method; (b) bit-shifts only method. In both cases $\mu_{\text{peakred}} = 0.98\mu_{\text{clip}}$.

this waveform, for example as proposed in [20], should give some further performance improvement. To simplify the notation, we define the *clip factor* and the *peak reduction factor* as

$$\mu_{\text{clip}} \stackrel{\text{def}}{=} \frac{\text{clip level}}{\sigma} \qquad \text{and} \qquad \mu_{\text{peakred}} \stackrel{\text{def}}{=} \frac{\text{peak reduction level}}{\sigma}$$

respectively.

4.1 Choosing the peak reduction factor

In our method, which has no iterations but still attempts to reduce more than one peak, one of the important choices is to what level the peaks should be reduced. It is desirable to aim for minimum frame clip rate. One solution would be to reduce the peaks down to the clip level, $\mu_{\text{peakred}} = \mu_{\text{clip}}$. This works if only one peak is reduced, but otherwise the scaled and shifted waveforms, c_1, c_2 and c_3, will interact with each other. This interference can recreate clips, although in general small such. Thus, to reduce most of the peaks below the clip level in only one iteration, the peak reduction level should be smaller than the clip level. However, if this safety margin is too wide, the waveforms and their side-lobes become larger. This increases the probability that the peak approximations "lift up" peaks that previously were below the clip level. Therefore, the frame clip rate is minimimum for some peak reduction level only slightly below the clip level.

Simulations indicate that the ratio $\mu_{\text{peakred}}/\mu_{\text{clip}}$ generally should be in the range 0.95–0.98. The performance is better than for the ratio 1, and (in our system) changes for the worse begin to be visible again around the ratio 0.93.

4.2 Performance

Let us then look at the improvements given by the PAR-reduction. Figure 3 (a) shows the frame clip rate[2] when the ratio $\mu_{\text{peakred}}/\mu_{\text{clip}} = 0.98$ and when true multiplications have been used to scale the peak-cancellation waveform. Curves are shown for one, two, three and five peaks reduced. At a frame clip rate of 10^{-4}, the simulations indicate that the clip level can be reduced 1.5 and 2.0 dB when we reduce one and two peaks, respectively. Only minor performance improvements follow from reducing more than 2 peaks, because of the low probability of several clips in one frame.

Figure 3 (b) shows the corresponding simulations when bit-shifts have been used instead of multiplications, as described in section 4.3. If the desired frame clip rate is 10^{-4}, then Figure 3 (b) suggests that the clip level can be reduced about 1.5 and 1.7 dB for one and two reduced peaks, respectively. This is close to the 1.5–2.0 dB in Figure 3 (a).

[2]The clip rate plots are directly comparable to Tellado's plots [20, 23, 24, 25], since μ_{clip} [dB] $= 20 \cdot \log_{10}(\mu_{\text{clip}}) = 10 \cdot \log_{10}(\mu_{\text{clip}}^2) = 10 \cdot \log_{10}(\text{PAR}_0) = \text{PAR}_0$ [dB]. Note, however, that our original signal has a higher PAR, since our system has 2048 carriers and Tellado's plots are for a system with 256 carriers.

4.3 Reducing complexity

The complexity of the original PAR-reduction device can be reduced considerably with only a small loss in performance. Here, we replace multiplications with bit-shifts, as described in Section 3. Rounding upwards to the nearest power of two[3] is similar to using a too low peak-reduction level, as discussed in Section 4.1. The question arises if our method is sensitive to these round-offs. However, the average height of the peak to be reduced is the center of gravity of the tail of the (almost) Gaussian amplitude distribution of the DMT-signal. For $\mu_{\text{clip}} = 4.5$ (13 dB in the plots), the expected clip amplitude is about 20 times smaller than the clip level. The average round-off error is then a few percent of the clip level, corresponding to using a peak-reduction level of, say, 0.95 instead of 0.98, resulting in only a minor performance loss.

Maybe the most important observation when it comes to complexity is that we do not use the IFFT to generate the waveform containing the approximate peak. (To do this, the FFT-processor would need to process every block more than once.) An important part of our current implementation is to precalculate the waveform, store it in a memory, and, with the right scaling and delay, blockwise and cyclically subtract it from the IFFT output.

With this, the following computations are needed to simultaneously reduce at most M peaks in a $2N$-sample DMT-frame:

- The first M peaks that would be clipped are found (if they exist). This does not require more than $2N$ *comparisons*.
- The $\leq M$ copies of the kernel are *bit-shifted* to get the desired amplitude and then *subtracted* from the original signal.

This adds up to at most $2N$ comparisons, $2MN$ additions, and $2MN$ bit-shifts. In the presented simulations we reduce the *largest* clips, not the first ones found; We do not expect much difference in performance, while we expect the latter to be easier to implement in practice.

5 Summary and conclusions

In this paper we suggest a low complexity PAR-reduction method based on a previously derived algorithm. We add implementation-related restrictions and refinements, which we found necessary for the implementation in our target VDSL-system. We also give some recommendations of parameter settings based on preliminary analyses discussed in the paper. One interesting result is that replacing multiplications with bit-shifts (multiplication with powers of two only) significantly decreases our method's complexity, but has a very small impact on its performance.

References

[1] ANSI T1E1.4. *Very-high-speed Digital Subscriber Lines. System Requirements.* VDSL SR: 98-043R5, draft technical document, revision 16, Sept. 1998.

[2] J. S. Chow, J. A. C. Bingham, and M. S. Flowers. Mitigating clipping noise in multi-carrier systems. In *IEEE international conference on communications*, volume 2, pages 715–719, 1997.

[3] A. Czylwik. Comparison between adaptive OFDM and single carrier modulation with frequency domain equalization. In *Proceedings of the 1997 47th IEEE Vehicular Technology Conference*, pages 865–869, Phoenix, AZ, May 1997. Also presented at COST 259 as TD(97)029.

[4] M. Friese. Multicarrier modulation with low peak-to-average power ratio. *Electronics Letters*, 32(8):713–714, Apr. 1996.

[5] A. Gatherer and M. Polley. Clip mitigation techniques for T1.413 Issue 3. ANSI document T1E1.4/97-397, Sacramento, CA, Dec. 1998.

[6] T. F. Hoo and V. K. Wei. Synthesis for low-crest waveforms for multicarrier CDMA system. In *CTMC proceedings of the 1995 4th communication theory mini-conference*, pages 131–135, Singapore, Nov. 1995.

[7] M. Isaksson, P. Deutgen, F. Sjöberg, S. K. Wilson, P. Ödling, and P. O. Börjesson. Zipper – a flexible duplex method for VDSL. In *Proceedings of the International Workshop on Copper Wire Access Systems (CWAS)*, pages 95–99, Budapest, Hungary, Oct. 1997.

[3] The bit-shifts can be combined with the suggestion in [23, 24] to store $\alpha_k \mathbf{p}$ for a few values of α_k. For example, if $\mathbf{p}$ and $\sqrt{2}\,\mathbf{p}$ are stored, then the bit-shifts lead to round-offs to powers of $\sqrt{2}$.

[8] X. Jin and S. Periyalwar. Real number TCM design for given peak-to-average power ratio. In *Proceedings of the 1994 Canadian Conference on Electrical and Computer Engineering*, volume 1, pages 222–225, Halifax, Canada, Sept. 1994.

[9] X. Li and J. A. Ritcey. M-sequences for OFDM peak-to-average power ratio reduction and error correction. *Electronics Letters*, 33(7):554–555, Mar. 1997.

[10] D. J. G. Mestdagh, P. M. P. Spruyt, and B. Biran. Effect of amplitude clipping in DMT-ADSL transceivers. *Electronics Letters*, 29(15):1354–1355, July 1993.

[11] S. H. Müller and J. B. Huber. A comparison of peak power reduction schemes for OFDM. In *Proceedings of the IEEE Global Telecommunications Conference (Globecom '97)*, pages 1–5, Phoenix, AZ, Nov. 1997.

[12] S. H. Müller and J. B. Huber. A novel peak power reduction scheme for OFDM. In *Proceedings of the international symposium on personal, indoor and mobile radio communications (PIMRC '97)*, pages 1090–1094, Helsinki, Finland, Sept. 1997.

[13] S. H. Müller and J. B. Huber. OFDM with reduced peak-to-average power ratio by optimum combination of partial transmit sequences. *Electronics Letters*, 33(5):368–369, Feb. 1997.

[14] S. Narahashi, K. Kumagai, and T. Nojima. Minimising peak-to-average power ratio of multitone signals using steepest descent method. *Electronics Letters*, 31(18):1552–1554, Aug. 1995.

[15] S. Narahashi and T. Nojima. New phasing scheme of N-multiple carriers for reducing peak-to-average power ratio. *Electronics Letters*, 30(17):1382–1383, Aug. 1994.

[16] S. Narahashi and T. Nojima. A new phasing scheme for multitone signal systems to reduce peak-to-average power ratio. *Electronics and Communications in Japan, Part 1*, 80(1):89–97, Nov. 1997.

[17] F. Sjöberg, M. Isaksson, P. Deutgen, R. Nilsson, P. Ödling, and P. O. Börjesson. Performance evaluation of the Zipper duplex method. In *Proceedings of the International Conference on Communication*, pages 1035–1039, Atlanta, GA, June 1998.

[18] F. Sjöberg, M. Isaksson, R. Nilsson, P. Ödling, S. K. Wilson, and P. O. Börjesson. Zipper – a duplex method for VDSL based on DMT. Submitted to *IEEE Transactions on Communication*, 1998.

[19] J. Tellado. *Multicarrier Transmission with low PAR*. Ph. D. thesis, Stanford University, CA, 1998.

[20] J. Tellado and J. M. Cioffi. PAR reduction in multicarrier transmission systems. ANSI document T1E1.4/97-367, Sacramento, CA, Dec. 1997.

[21] J. Tellado and J. M. Cioffi. Further results on peak-to-average ratio reduction. ANSI document T1E1.4/98-252, San Antonio, TX, Aug. 1998. Also presented at ETSI TM6 meeting in Vienna, Austria, Sept. 1998.

[22] J. Tellado and J. M. Cioffi. PAR reduction with minimal or zero bandwidth loss. ANSI document T1E1.4/98-173, Huntsville, AL, June 1998.

[23] J. Tellado and J. M. Cioffi. Peak power reduction for multicarrier transmission. In *Communications Theory Mini-Conference, Globecom '98*, Sydney, Australia, Nov. 1998.

[24] J. Tellado and J. M. Cioffi. Revisiting DMT's PAR. ANSI document T1E1.4/98-083, Austin, TX, Mar. 1998.

[25] J. Tellado and J. M. Cioffi. Revisiting DMT's Peak-to-Average Ratio. ETSI document TM6:TD08, Antwerp, Belgium, Apr. 1998.

[26] J. Tu and J. M. Cioffi. A loading algorithm for the concatenation of coset codes with multichannel modulation methods. In *Proceedings of the IEEE Global Telecommunications Conference (Globecom '90)*, pages 1183–1187, San Diego, CA, Nov. 1990.

[27] P. Van Eetvelt, G. Wade, and M. Tomlinson. Peak to average power reduction for OFDM schemes by selective scrambling. *Electronics Letters*, 32(21):1963–1964, Oct. 1996.

[28] R. D. J. van Nee. OFDM codes for peak-to-average power reduction and error correction. In *Proceedings of the IEEE Global Telecommunications Conference (Globecom '96)*, volume 1, pages 740–744, London, UK, Nov. 1996.

[29] D. Wulich. Reduction of peak to mean ratio of multicarrier modulation using cyclic coding. *Electronics Letters*, 32(5):432–433, Feb. 1996.